Introductory module: a video introduction to Modellus

Sample Workshops

ITforUS

(Information Technology for Understanding Science)

Education and Culture
Socrates

IT for US
Information Technology for Understanding Science
Socrates Comenius programme
Action 2.1 - Training of School Education Staff
119001-CP-1-2004-1-PL-COMENIUS-C21

I. Introduction

This module has 10 video introductions to Modellus, illustrating most of its functionalities.

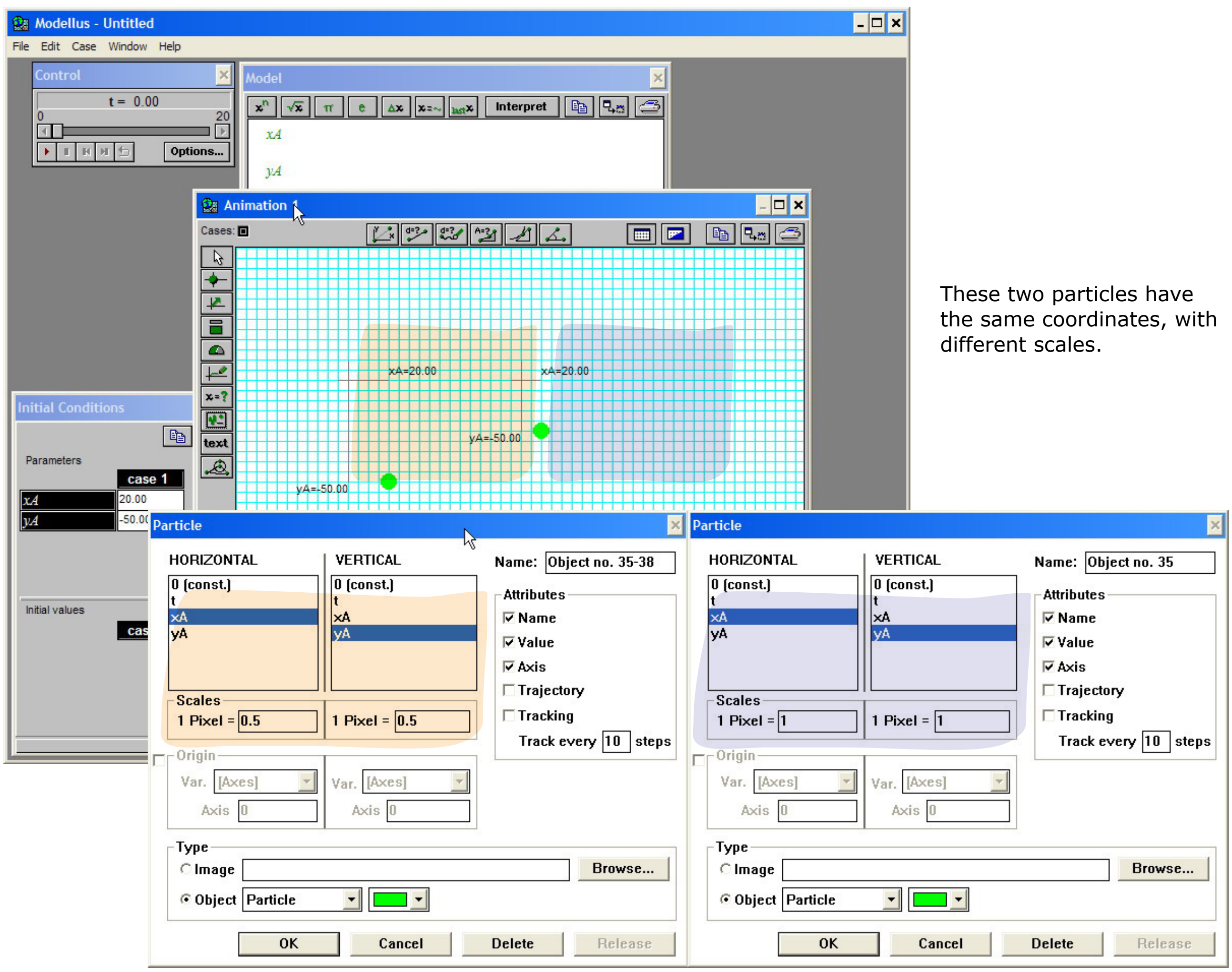
1. Background Theory

Cartesian coordinates on a plane

In a plane, any point can be used to define an origin. An origin is a particular point where two perpendicular axes can start.

Each axis is a straight line with an orientation. Defining a unit of distance on the axis, any point on the axis can have a real number associated with it. And each point on the plane can be identified by two numbers, the cartesian coordinates, one in each axis.

Modellus use the screen pixel as unit: by default, 1 pixel = 1 unit. This can easily be changed for all type of objects on Modellus.

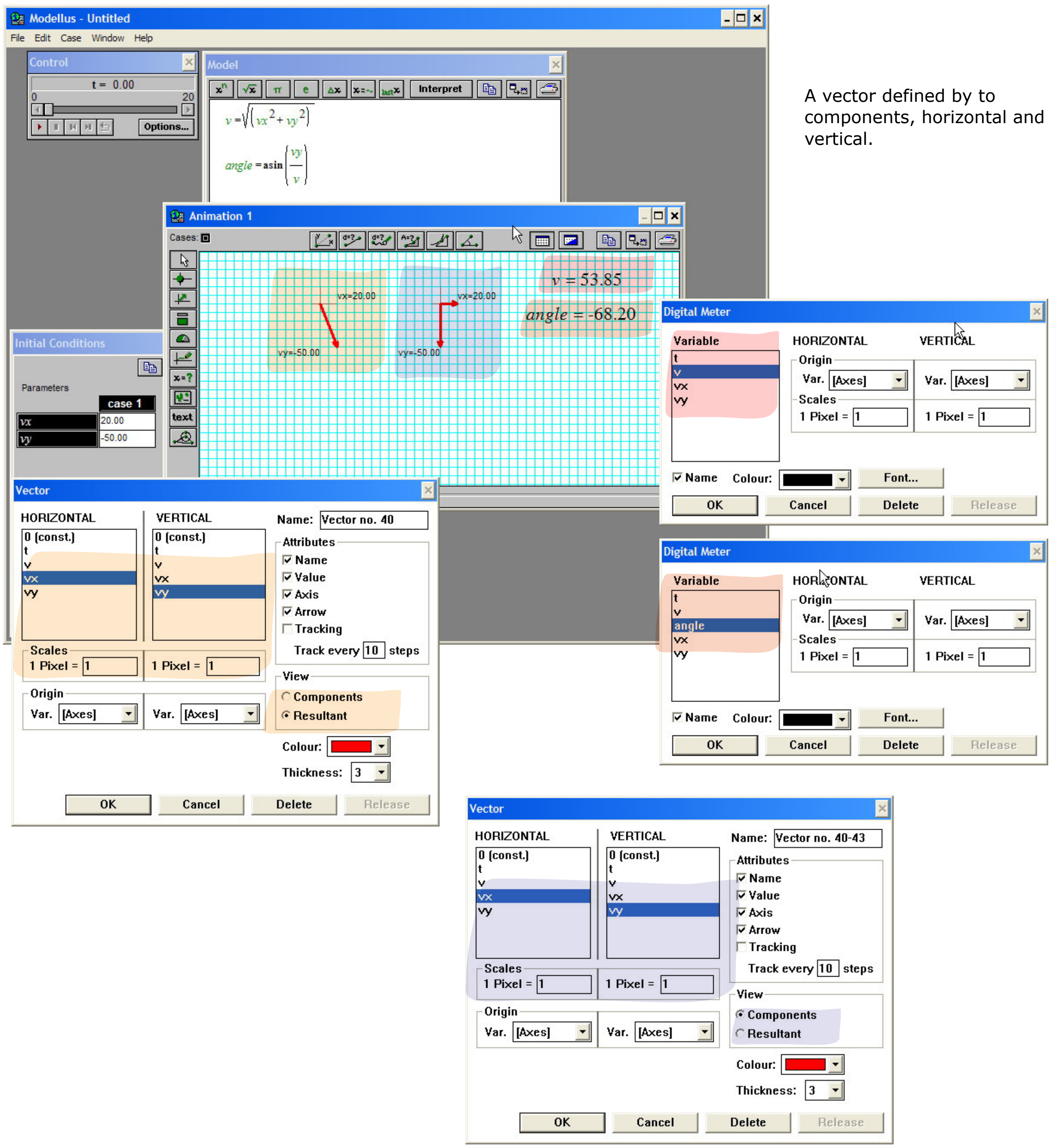


These two particles have the same coordinates, with different scales.

Vectors

Vectors are mathematical entities that have direction and magnitude. Vectors can be defined using vector components. To each vector component, there is a scalar component associated.

Vectors in Modellus are defined using two scalar components. From the components, it is possible to compute the direction, expressed as an angle with any of the axis, and the magnitude.



A vector defined by to components, horizontal and vertical.

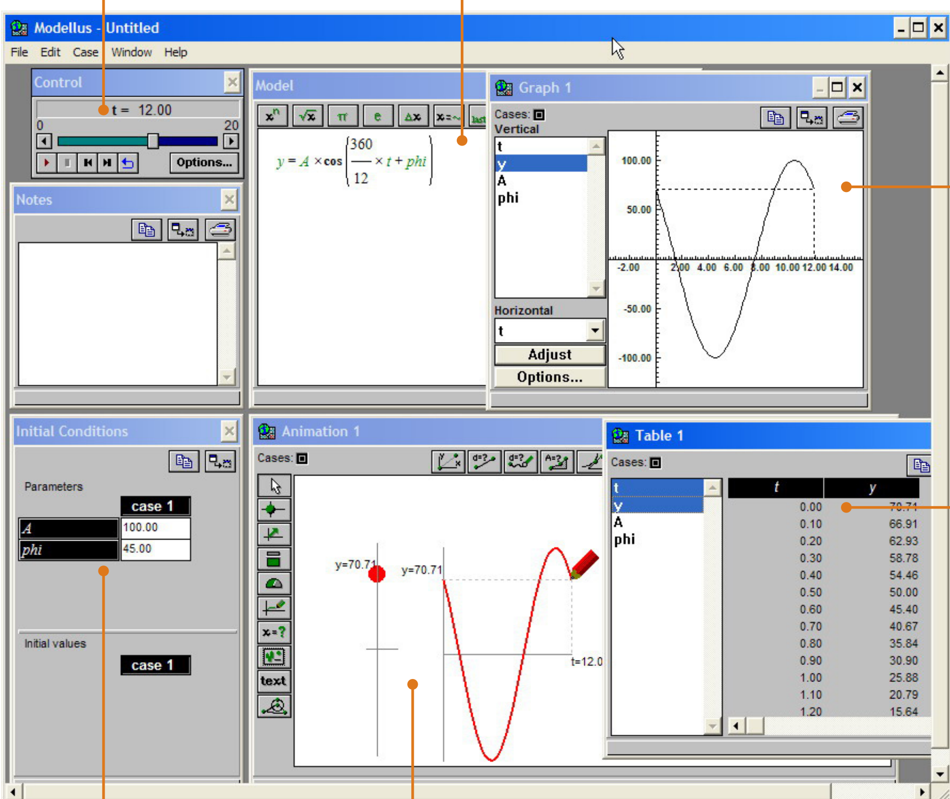
Variables, functions, tables and graphs

Variables express properties of objects, space, etc. Most physical variables can be expressed as quantities, using real numbers and units.

Some variables are dependent of the values of other variables. A function expresses a relation between dependent and independent variables. By definition of function, for each value of the independent variable, there is only one value of the dependent variable.

The relation between variables can be represented on a word statement, on a table of values, on a graph, or on a mathematical expression. Expressing functions as mathematical expressions is, usually, the most synthetic way of expressing a relation between variables.

Scientific software, and Modellus in particular, can be very useful for linking these multiple ways of representing functions and other mathematical objects.



Oscillations, functions and differential equations

Sine and co-sine functions are used to describe harmonic oscillations. An harmonic oscillation is characterized by a period (or a frequency). The later two are determined by the initial conditions.

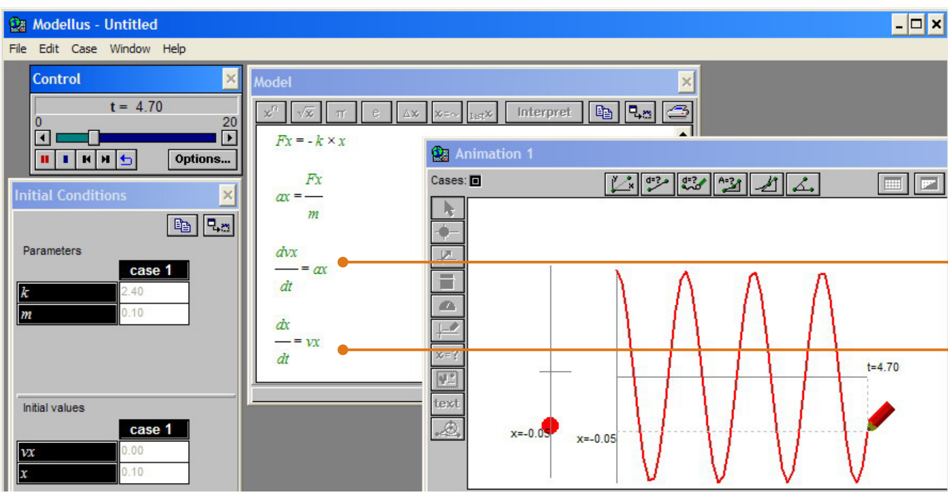
Modellus can use both functions and numerical integration to investigate oscillations. For example, the user can write a sine or co-sine function to describe an oscillation (position, velocity, etc.) or can use Hooke's law (the magnitude of the restoring force on the oscillator is proportional to the displacement) to compute acceleration, then integrate acceleration to compute velocity and finally use velocity to compute position.

To integrate a variable, the user must define a rate of change. The integrated variable is the accumulated change, computed from the rate of change. By default, Modellus uses a time step of 0.1 units and the Runge-Kutta 4th order method.

Integration (i.e., accumulated change), can also be computed using iterative equations, such as those that express Euler or Euler-Cromer methods.

$$y = A \times \cos\left(\frac{360}{12} \times t + phi\right)$$

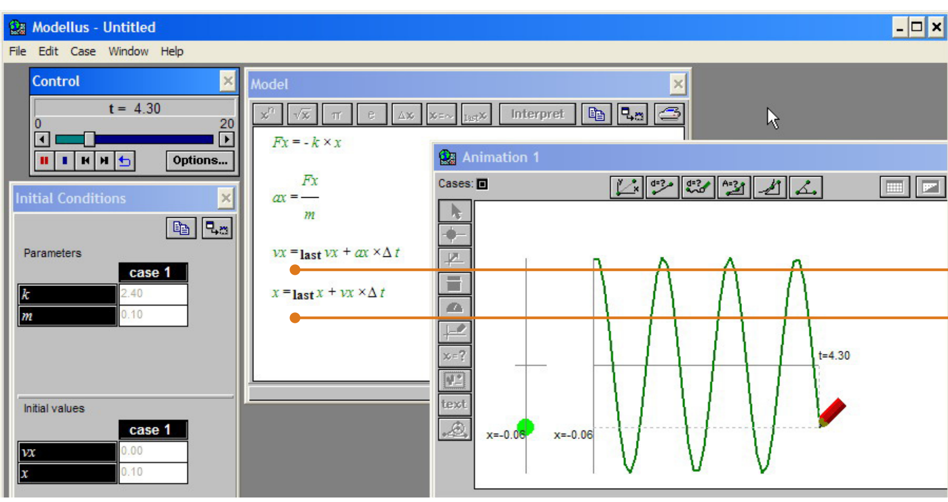
An oscillation can be obtained with a function, as shown on the previous page...



An oscillation can also be obtained with differential equations...

The instantaneous rate of change of the velocity is the acceleration...

The instantaneous rate of change of the position is the velocity...



An oscillation can also be obtained with iterative equations...

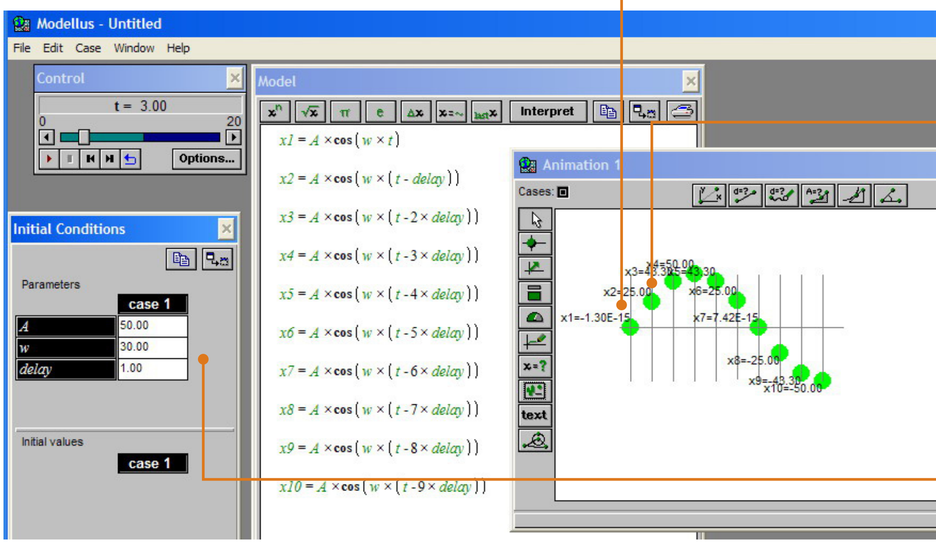
Velocity accumulates from the previous value and the change equals the rate of change (acceleration) multiplyed by the time step......

Position accumulates from the previous value and the change equals the rate of change (velocity) multiplied by the time step......

Waves

A wave is a change in one or more physical properties through space and time, around a reference value.

Certain types of waves can be described using sinusoidal functions. In each point, there is a quantity that oscillates periodically and in consecutive points this quantity oscillates with a delay from the previous point.



This particle oscillates...

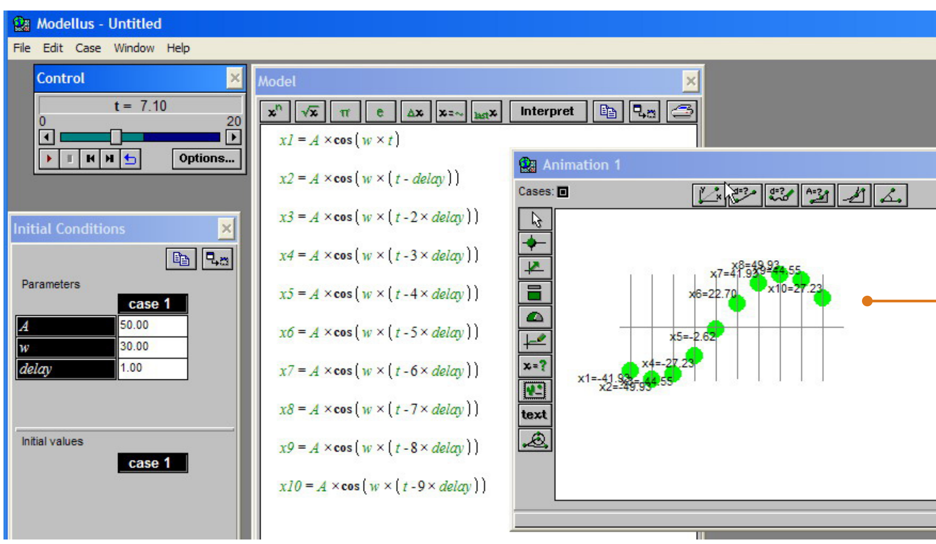
The second particle oscillates with a certain delay relative to the first particle...

The third particle oscillates with twice the delay...

Etc...

Amplitude and frequency for each oscillation

Delay of the oscillation of the second particle



The combined oscillations of all particles is a transverse wave (the direction of oscillations is perpendicular to the propagation direction of the wave)

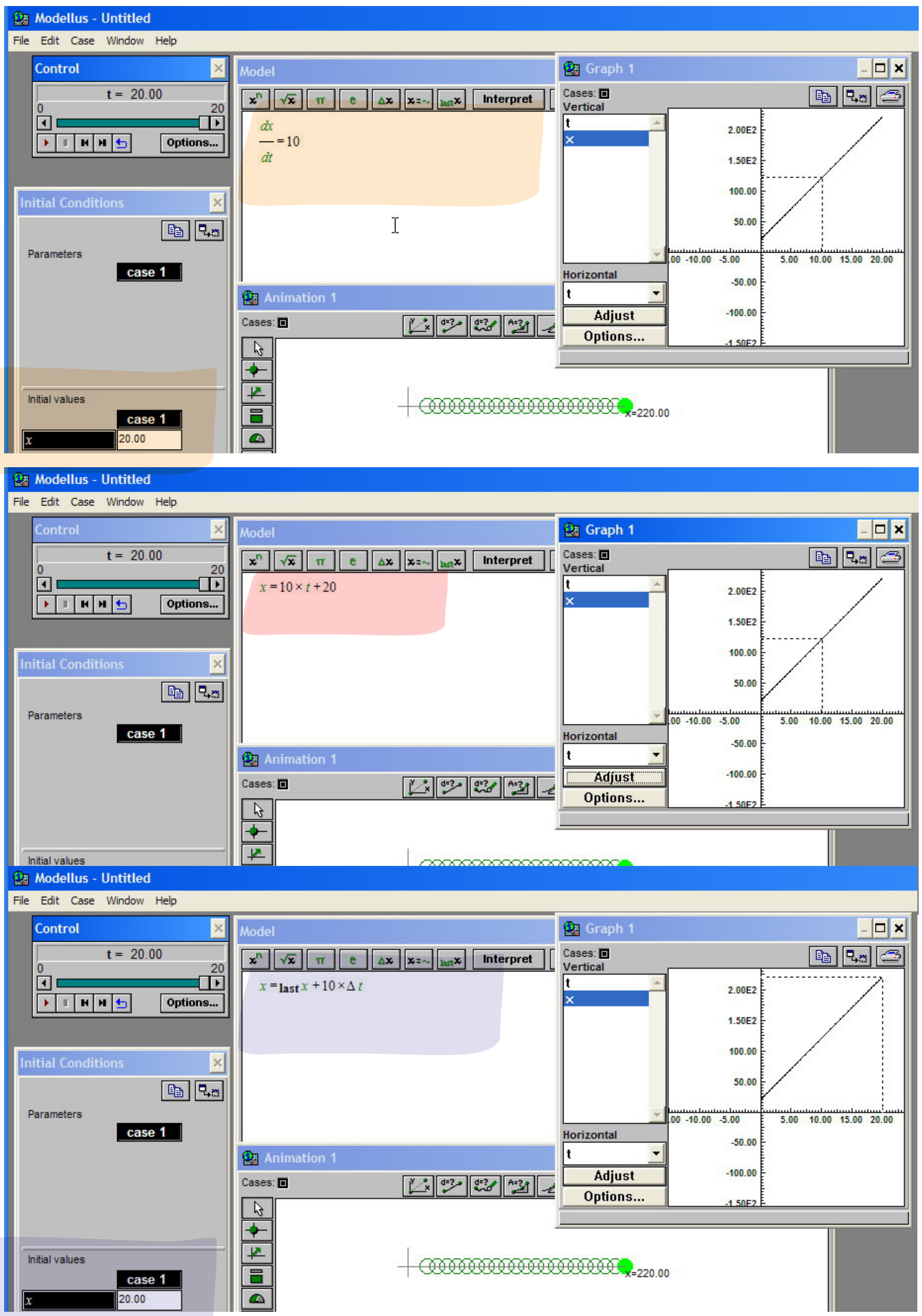
Differential equations and functions

A function such as $x = 10\ t$ states that the rate of change of x is 10 units: every time the variable t changes 1 unit, the variable x changes 10 units.

This relation can also be represented as a differential equation, $dx/dt = 10$. This equation can be read as "the instantaneous rate of change of x is 10 units". To start computing x, the software needs an "initial value for x": it knows the rate of change, so it needs to know from which value the variable starts changing…

Modellus also accepts iterative equations to represent change. For example, the previous differential equation $dx/dt = 10$ can be represented as x = last(x) + rate of change × time step. This iterative computation gives the same values as the function $x = 10\ t$ or the differential equation $dx/dt = 10$, since the rate of change is constant. When the rate of change is not constant, the values given by the iterative equation approximate the exact and correct values as the time step tends to smaller values.

Any function of the type $x = 10\ t$ + *constant* is called an analytical solution of the differential equation $dx/dt = 10$, where *constant* is the initial value for x.

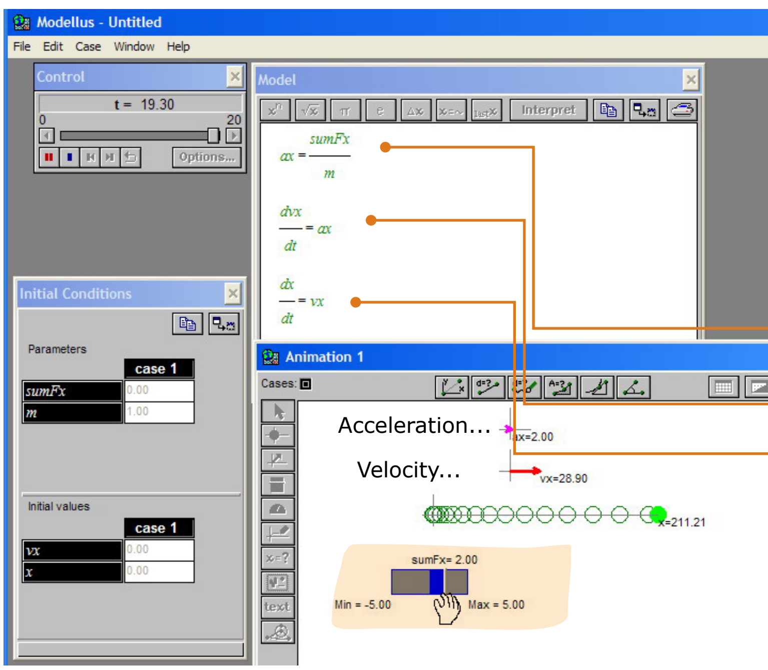


Differential equations and inertia

Newton's theory states that velocity is the instantaneous rate of change of the position vector (a vector that has a tail on the origin of the reference frame and the head on the current position). For one dimensional motion, this can be stated as "the scalar component of velocity, v_x, is the instantaneous rate of change of x". Similarly, acceleration is the instantaneous rate of change of the velocity. Again, for motion in one dimensional motion, we can say that "the scalar component of the acceleration, a_x, is the instantaneous rate of change of the velocity, v_x".

Modellus computes solutions of differential equations using a numerical method. This means that, for example, in the case constant acceleration, it doesn't know that the solution for the function x is ½ $a_x t^2$. But the method used by Modellus is powerful enough to give a numerical solution that is equal or almost equal to the analytical solution in this case and in many more cases. The basic idea in solving differential equations is that a new value of a variable (lets' say x) is computed using an algorithm of the type new value = old value + rate of change × time step. If the time step is conveniently small and the "rate of change" is known, this iterative computation can compute new values of the variable as time goes on.

The advantages of using numerical methods to solve differential equations are somewhat evident: controlling the rate of change, the user can control how something moves on the screen.



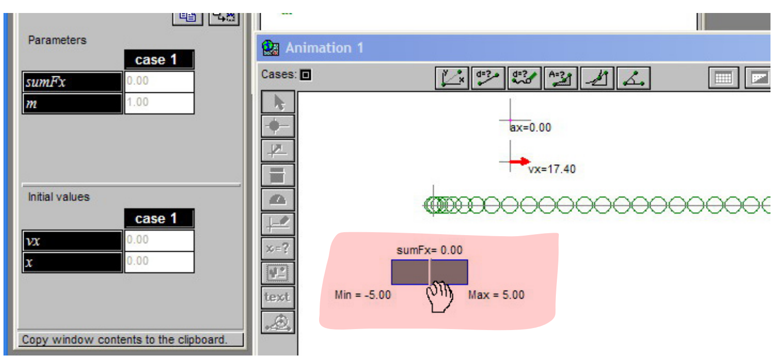
This model can illustrate how inertia is described mathematically. Assign the value 0 to all parameters except mass (which can be 1) and assign to the Level Indicator the properties highlighted. Run the model and, after a few seconds, drag the *sumFx* Level Indicator to the right. The particle starts moving, accelerating. Then move again the slider to 0... (net force *sumFx* becomes zero) and you will see that the particle continues moving, unless you change the net force again.

Newton's second law: how to compute acceleration, the rate of change of velocity

The rate of change of velocity is acceleration...

The rate of change of position is velocity...

Particle accelerating to the right (net force points to the right)...



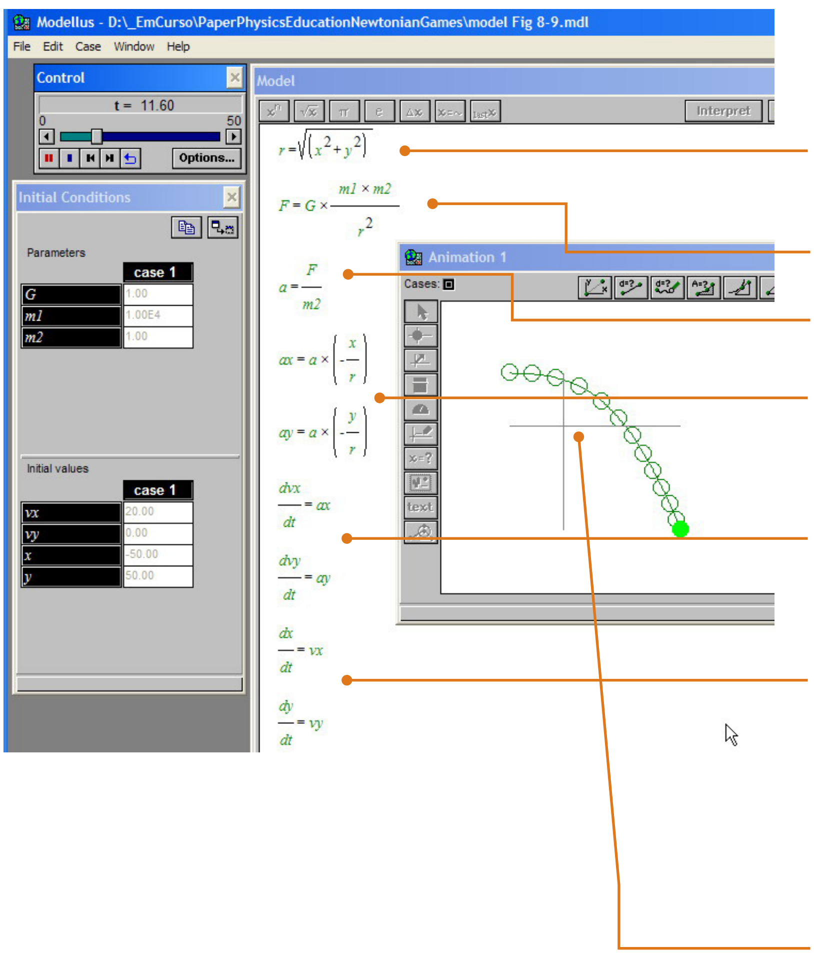
Particle moving with constant velocity, after being accelerated to the right (during acceleration, net force points to right; then no net force)...

Universal gravitation and differential equations

Newton's law of gravitation establish how to compute the gravitational force between objects, knowing their mass and the distance between them. If one of the objects (a planet...) has a large mass m_1 and is placed on the origin of a reference frame xOy, one can assume that only an object with a small mass (a spaceship, a satellite, anything you want…), m_2, will be accelerated. Object 2 will travell on the gravitational field created by object 1, acted by the gravitational force.

To make a mathematical model of this interaction, we need to start by computing the distance r between the two particles, then the magnitude of the gravitational force and the magnitude of the acceleration of particle m_2. This acceleration is then decomposed in the two components, x and y, using simple trigonometric relations (cosine and sine) and taking into account that the forces are attractive (then the minus sign on the scalar component—acceleration and force point towards the origin of the reference frame). Once we have the acceleration components, we can write that the rate of change of velocity is acceleration and that the rate of change of position is velocity (in the two dimensions).

As can been seen from the Initial Conditions window, the gravitational constant G has an arbitrary value of 1 (just to simplify things), mass m_1 is 10000 times bigger than mass m_2 (that's why we can neglect the gravitational force on m_1), the initial position of mass m_2 is (-50, 50) and it is moving on the Ox direction with a initial speed of 20 units.



Distance between objects (the more massive object is placed on the origin of the referential)

Newton's law of gravitation

Computing the magnitude of the acceleration of the less massive object

Computing the components of the acceleration

The instantaneous rate of change of velocity is acceleration...

The instantaneous rate of change of position is velocity...

The more massive object is placed on the origin of the reference frame

Chemical kinetics, chemical equilibrium and differential equations

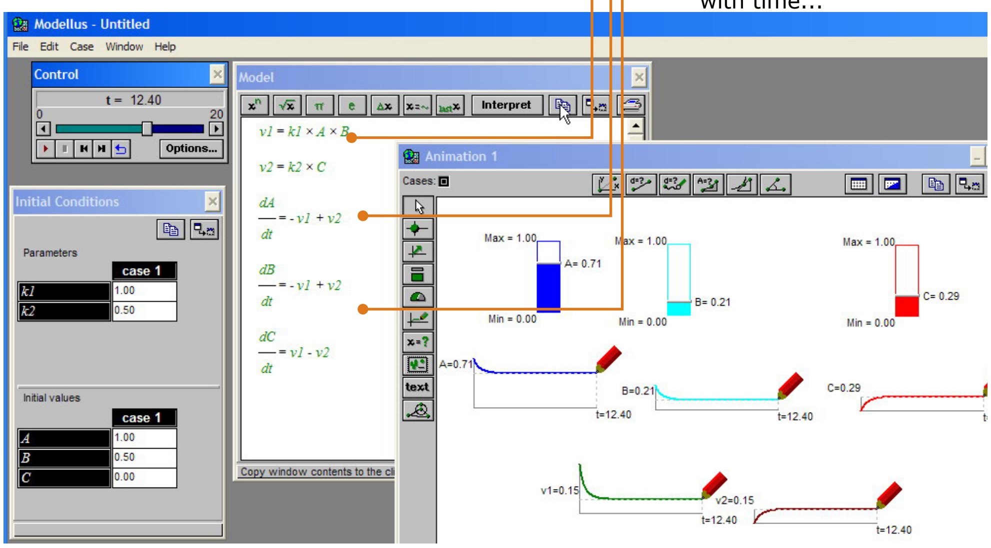
Many more dynamic phenomena can be modelled with simple differential equations. Another example used in this module is the kinetics of chemical reactions.

A fundamental concept in chemical kinetics is the rate law. A rate law is a mathematical expression of how the rate of reaction depends on the concentrations (or other physical properties) of chemical species.

For a generic reaction A + B → C, the simplest rate equation has the form:

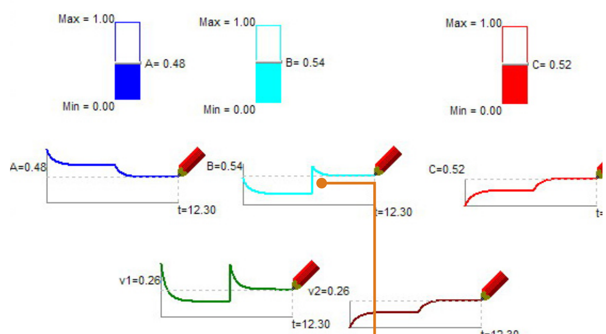
$$v = k\ A^{a}\ B^{b}$$

In this equation, A and B expresses the concentration of species A and B, usually in mol/litre, and k is the reaction rate coefficient or rate constant (not really a constant, because it includes everything that affects reaction rate besides concentration, such as temperature). The exponents a and b are called reaction orders and depend on the reaction mechanism.
The stoichiometric coefficients in the chemical equation and the reaction orders are often the same. Complicated rate equations can have a sum of terms or even have quantities in denominators.



Rate equations are differential equations and can be integrated, numerically or analitically, in order to obtain functions that relate concentrations with time.

For reversible chemical reactions, chemical equilibrium is the state in which the chemical activities or concentrations of the species do not change over time. Generally, this happens when the forward chemical process proceeds at a rate equal to their reverse reaction. At equilibrium, reaction rates being equal, there are no net changes in any of the species (the equilibrium is dynamic).



Le Chatelier's principle is used to predict how the system reacts when there are changes in the species (or its properties) during equilibirum: after the change, the equilibrium shifts to partially counter-act the imposed change. In the case of the example on this page, increasing the concentration of species B, forces the system to decrease this species and increase the other species A and C. After the change, reaction rates become equal again.

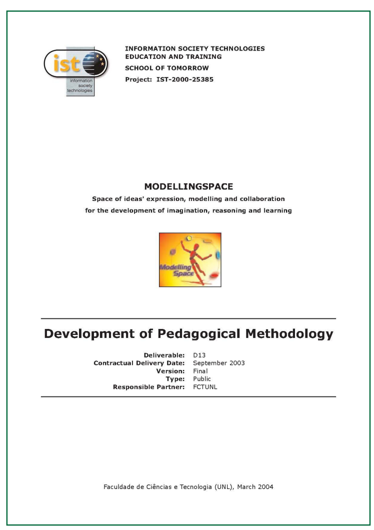
2 Science concepts introduced in this module

This module uses concepts from Physics, Mathematics, and Chemistry and assumes the reader (a teacher or a teacher educator) is relatively familiar with them. The goal of the module is to **illustrate in a concrete way** concepts, principles and ideas, such as:

- Reference frame, origin of a reference frame and coordinates in a plane;
- Independent and dependent variables;
- Functions and graphs;
- Scales;
- Vectors and vector components;
- Speed, velocity, and acceleration;
- Newton's laws of motion;
- Linear, quadratic and sinusoidal functions;
- Oscillations and Hooke's law;
- Waves;
- Differential equations, analytical and numerical integration;
- Universal gravitation;
- Chemical kinetics and equilibria.

3. Other information

Annexed to this module is a text on a pedagogical methodology for using modelling in schools. A pedagogical methodology is a set of procedures that a teacher can develop in order to help all students learn. A methodology is seen as something one cannot receive from others. On the contrary, it is the complex result of instruction, personal experience and reflection. The methodology establish a framework for the procedures, based on six tenets (Commitment to teaching, to students and to their learning; knowledge of science and mathematics; knowledge of students; knowledge of the art of teaching; science as a way of thinking; and reflection and professional growth) and makes thirteen proposals: (1) make clear goals and plan how concepts and ideas evolve during the activities, anticipating learning difficulties; (2) elicit and verbalize students' conceptions; (3) promote interaction, collaboration, and group cohesion; (4) give prompt feedback; (5) induce self and group formative assessment; (6) proceed from concrete to abstract; (7) verbalize mathematical procedures; (8) promote schematic drawing and writing as "tools-to-think-with"; (9) scaffold the transition from direct computations to algebraic reasoning, from number sense to symbol sense; (10) explore multiple representations; (11) make abstract objects as concrete as possible but spot the differences between the "real thing" and the representation; (12) balance exploratory learning with guided learning; (13) anticipate, check, and revise the coherence of the model and data.


INFORMATION SOCIETY TECHNOLOGIES
EDUCATION AND TRAINING
SCHOOL OF TOMORROW
Project: IST-2000-25385

MODELLINGSPACE
Space of ideas' expression, modelling and collaboration
for the development of imagination, reasoning and learning

Development of Pedagogical Methodology

Deliverable: D13
Contractual Delivery Date: September 2003
Version: Final
Type: Public
Responsible Partner: FCTUNL

Faculdade de Ciências e Tecnologia (UNL), March 2004

II. Didactical approach

1. Pedagogical context

The activities presented in this module can be used by teachers and teacher trainers, in Physics and Chemistry but also in Mathematics.

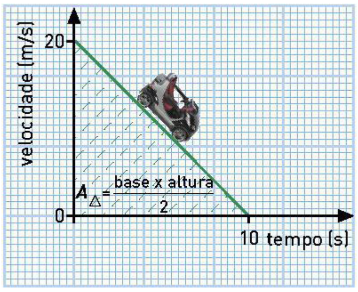
They were not designed to fit in any training curriculum. They simple illustrate how Modellus can be used to model and explore different phenomena.

2. Common difficulties

Physics, chemistry, and mathematics education research has consistently verified that very often students and teachers alike have difficulties in the understanding of most of the concepts, ideas and laws used in the workshops.

Even in schools books we can find misconceptions, as the one shown on the right. Particular attention should be given to meaning and content of visual representations, such as graphs, trajectories, vectors, etc. Learners must always be attentive to the meaning of the representation, not on the "form of the drawing".

Instantaneous rates of change, differential equations and integration must be treated mainly as "concepts to work with", not as symbolic manipulation. The examples and the introduction of this module illustrates how this can be done.



An example of a common misconception found in a school book: the function — a physical quantity as a function of time — is confused with a trajectory for the car...

3. Evaluation of ICT

Computers are now the most common scientific tool, used in almost all aspects of the scientific endeavour, from measuring and modelling to writing and synchronous communication. It should then be *natural* to use computers in learning science.

Computers can be particular useful for learning **dynamic representations**, such as graphs and functions, because they allow the user to **explore multiple representations simultaneously**. But this *is not necessarily a factor of success in learning because learners can become confused with too many simultaneous representations*. **Careful teacher guidance** is essential to sense making of multiple representations: learners need to be guided in the process of **verbalization** of visual and algebraic representations and in the process of **linking multiple representations** of the same phenomenon.

4. Teaching approaches

Good classroom organization is an essential component in a successful teaching approach, particularly when using complex tools such as computers and software. Most approaches to classroom organization that can give good results **mix features** of **students' autonomous work**, both individually and in small groups, to **teacher lecturing** to all class.

Typically, teachers can start with an all class approach, with students following the lesson with a screen projector. It is almost always a good idea to *ask one or more students to work on the computer connected to the projector*. This allows the teacher to have direct information of students' difficulties when manipulating the software and to be slower on the explanation of the ideas and activities that are being presented.

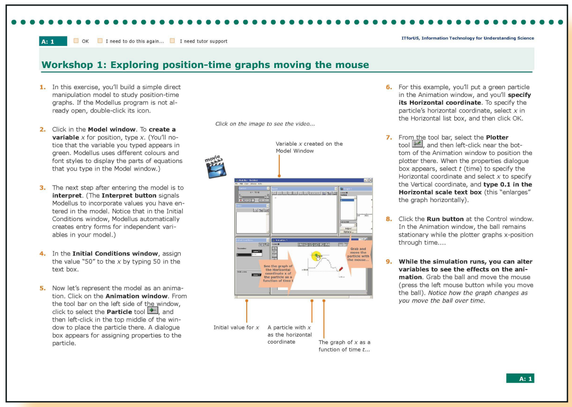
As all teachers know by experience, it is usually difficult for most students to follow written instructions, even when these instructions are only a few sentences long. To overcome this difficulty, teachers can ask students to **read the activities before** starting them and then promote a collective or group **discussion about what is supposed to be done with the computer**. *As a rule of thumb, students should only start an activity when they know what they will do on the activity: they will only consult the written worksheet just for checking details, not for following instructions.*

III. Activities / Workshops

Workshop 1: Exploring position-time graphs moving the mouse

On this workshop you can find:

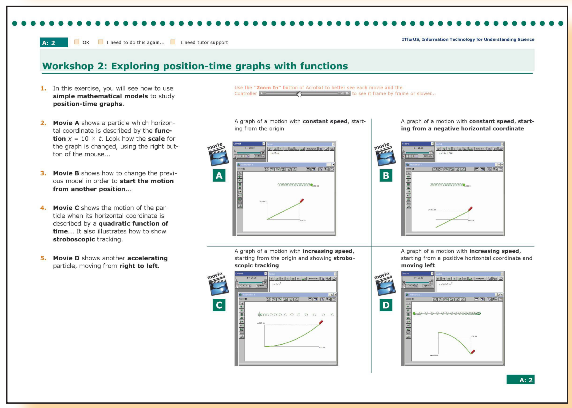
- Introduction to the basic manipulation of equations and how to animate objects in Modellus;
- Once the video is clicked (the image on the centre), a large video window is shown; this and the other videos has no sound (this is more appropriate to computers rooms with many computers).



Workshop 2: Exploring position-time graphs with functions

On this workshop you can find:

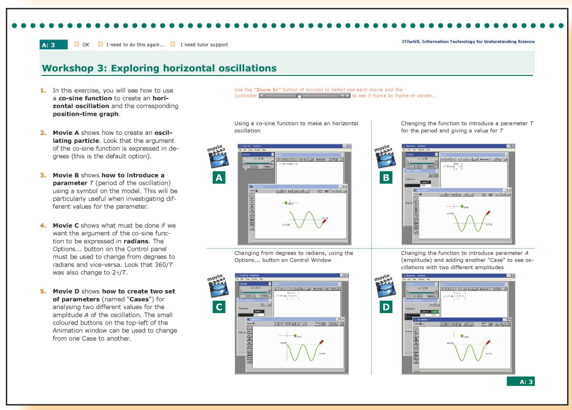
- How to create models with linear and quadratic functions;
- This page and the following pages has 4 videos; to see each with a proper size, it is necessary to use the zoom in function of Adobe Acrobat.



Workshop 3: Exploring horizontal oscillations

On this workshop you can find:

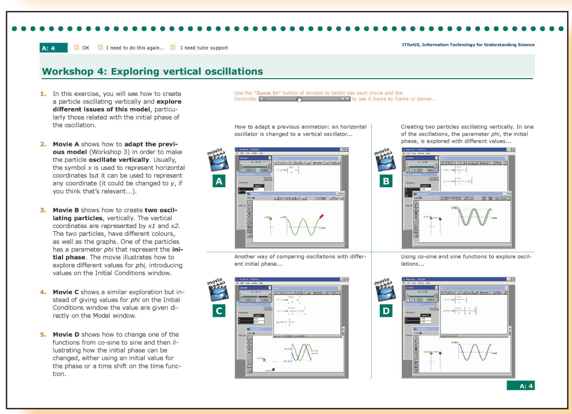
- An exploration of sinusoidal functions, using parameters.



Workshop 4: Exploring vertical oscillations

On this workshop you can find:

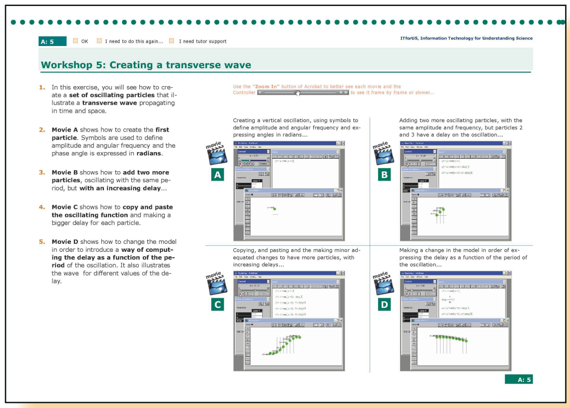
- More explorations of sinusoidal functions.



Workshop 5: Creating a transverse wave

On this workshop you can find:

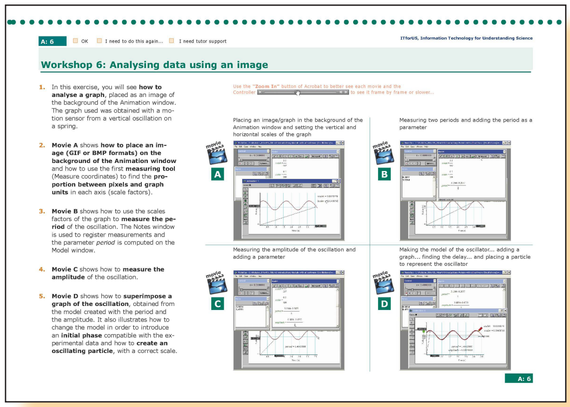
- How to create a transverse wave with oscillating particles.



Workshop 6: Analysing data using an image

On this workshop you can find:

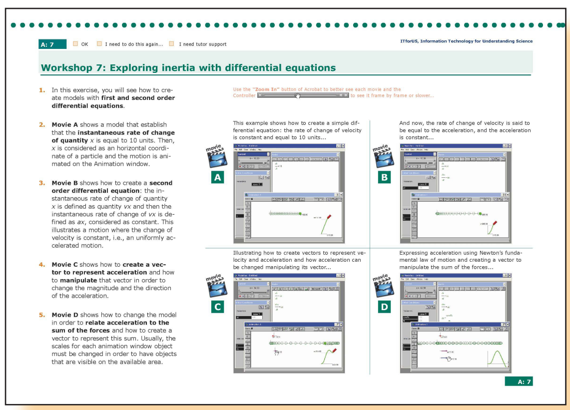
- How to use Modellus tools to analyze images and make models from the data on images.



Workshop 7: Exploring inertia with differential equations

On this workshop you can find:

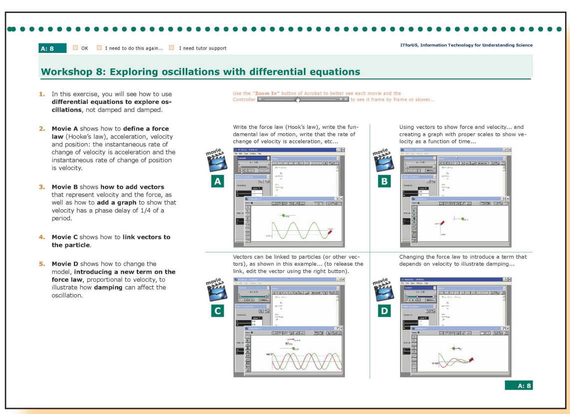
- How to explore Newton’s theory of motion, using instantaneous rates of change and differential equations.



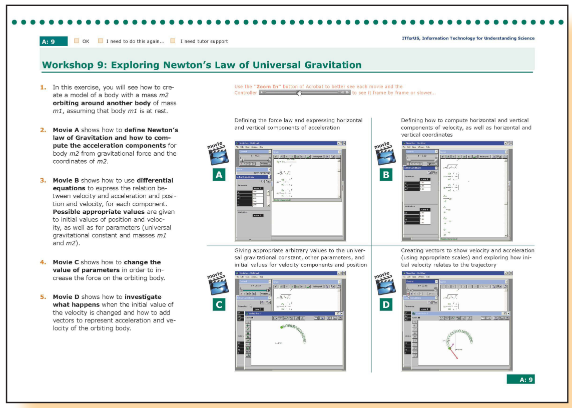
Workshop 8: Exploring oscillations with differential equations

On this workshop you can find:

- How to create a model of an oscillator using Hooke’s law and differential equations.



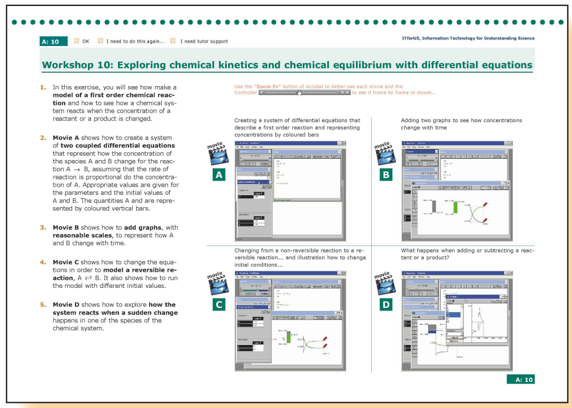
Workshop 9: Exploring Newton’s Law of Universal Gravitation



On this workshop you can find:

- How to create a model that illustrates Newton’s Law of Universal Gravitation.

Workshop 10: Exploring chemical kinetics and chemical equilibrium with differential equations



On this workshop you can find:

- How to create a model that illustrates the kinetics of chemical reactions, including change in the species once they reach equilibrium and what does dynamic equilibrium means.